EXERCISE 6 (1)

B.Sc. Math (H) D1(H) Part-I
Paper-I, Group-A
Summation of series
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1 (iv) Prove $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11} = \frac{1}{2}$

Here $\alpha = \frac{\pi}{11}$, $\beta = \frac{2\pi}{11}$, $n = 5$

$$S = \frac{\sin\frac{5\times 2\pi}{2\times 11}}{\sin\frac{2\pi}{2\times 11}} \cos\left\{\frac{\pi}{11} + (5-1)\frac{2\pi}{2\times 11}\right\}$$

$$= \frac{\sin\frac{5\pi}{11}}{\sin\frac{\pi}{11}} \cos\left\{\frac{\pi}{11} + \frac{4\pi}{11}\right\}$$

$$= \frac{2\sin\frac{5\pi}{11}\cos\frac{5\pi}{11}}{2\sin\frac{\pi}{11}}$$

$$= \frac{\sin 2\times\frac{5\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{\sin\frac{10\pi}{11}}{2\sin\frac{\pi}{11}}$$

$$= \frac{\sin\left(\pi - \frac{\pi}{11}\right)}{2\sin\frac{\pi}{11}} = \frac{\sin\frac{\pi}{11}}{2\sin\frac{\pi}{11}} = \frac{1}{2} \text{ proved}$$

1 (v) $\sin\frac{\pi}{14} + \sin\frac{2\pi}{14} + \sin\frac{3\pi}{14} + \dots$ to 28 terms $= 0$

Here $\alpha = \frac{\pi}{14}$, $\beta = \frac{\pi}{14}$, $n = 28$

अब सरल करें।

1 (vi) $\sin\frac{2\pi}{n} + \sin\frac{4\pi}{n} + \sin\frac{6\pi}{n} + \dots + \sin\frac{2n\pi}{n} = 0$ when $n > 2$

Here $\alpha = \frac{2\pi}{n}$, $\beta = \frac{4\pi}{n} - \frac{2\pi}{n} = \frac{2\pi}{n}$

$n = n$

Proceed as 1.

2 (i) $\sin^2\alpha + \sin^2 2\alpha + \sin^2 3\alpha + \cdots$ to n terms. (2)

Square को हटाना है तो दुगुना करके पर Formula को आधा, ऐसा Square हटा।

Let $S = \sin^2\alpha + \sin^2 2\alpha + \cdots$

$\therefore 2S = 2\sin^2\alpha + 2\sin^2 2\alpha + \cdots$

$2S = (1-\cos 2\alpha) + (1-\cos 4\alpha) + (1-\cos 6\alpha) + \cdots$

$= \{1+1+1+\cdots \text{ to n terms}\} - \{\cos 2\alpha + \cos 4\alpha + \cos 6\alpha \cdots \text{ to n terms}\}$

$$2S = n - \frac{\sin n\frac{2\alpha}{2}}{\sin\frac{2\alpha}{2}} \cos\left\{2\alpha + (n-1)\frac{2\alpha}{2}\right\}$$

$$\therefore S = \frac{n}{2} - \frac{\sin n\alpha}{2\sin\alpha}\cos\{2\alpha + (n-1)\alpha\}$$

$= 2\alpha + n\alpha - \alpha$ $(\alpha + n\alpha)$ $(n+1)\alpha$

2 (iii) $\cos^2\alpha + \cos^2(\alpha+\beta) + \cos^2(\alpha+4\beta) + \cdots$

Let $S = \cos^2\alpha + \cos^2(\alpha+\beta) + \cos^2(\alpha+4\beta) + \cdots$ to n terms

$\therefore 2S = 2\cos^2\alpha + 2\cos^2(\alpha+\beta) + 2\cos^2(\alpha+4\beta) + \cdots$ to n terms

$= (1+\cos 2\alpha) + \{1+\cos 2(\alpha+\beta)\} + \cdots$

$= (1+1+1+\cdots \text{ to n terms}) + (\cos 2\alpha + \cos(2\alpha+4\beta) + \cos(2\alpha+4\beta) + \cdots\}$

$$2S = n + \cancel{\cos}\ \frac{\sin n\frac{2\beta}{2}}{\sin\frac{2\beta}{2}} \cdot \cos\left\{2\alpha + (n-1)\frac{4\beta}{2}\right\}$$

$$\therefore S = \frac{n}{2} + \frac{1}{2}\frac{\sin n\beta}{\sin\beta}\cos\{2\alpha + (n-1)\beta\}$$

3 (i) $\sin^3\alpha + \sin^3(\alpha+\beta) + \sin^3(\alpha+2\beta) + \cdots$ to n terms

$S = \sin^3\alpha + \sin^3(\alpha+\beta) + \sin^3(\alpha+2\beta) + \sin^3(\alpha+3\beta) + \cdots$

$4S = 4\sin^3\alpha + 4\sin^3(\alpha+\beta) + 4\sin^3(\alpha+2\beta) + \cdots$

concept cube को हटाना है अर्थात 4 से गुणा किया।

$= (3\sin\alpha - \sin 3\alpha) + \{3\sin(\alpha+\beta) - \sin 3(\alpha+\beta)\}$

$+ \{3\sin(\alpha+2\beta) - \sin 3(\alpha+2\beta)\} + \cdots$ to n terms

$= \left[3\{\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \cdots\}\right]$

$- \left[\sin 3\alpha + \sin(3\alpha+3\beta) + \cdots \text{ to } n \text{ terms}\right]$

$$4S = 3\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left\{\alpha + (n-1)\frac{\beta}{2}\right\}$$

$$- \frac{\sin n\frac{3\beta}{2}}{\sin\frac{3\beta}{2}}\sin\left\{3\alpha + (n-1)\frac{3\beta}{2}\right\}\Big]$$

$S = \frac{1}{4}\left[\qquad\right]$ Ans

$\sin 3\alpha = 3\sin\alpha - 4\sin^3 A$

$\therefore 4\sin^3 A = 3\sin\alpha - \sin 3\alpha$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\cos 3A + 3\cos A = 4\cos^3 A$

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